Policy irreversibility and interest rate smoothing

Teruyoshi Kobayashi*

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Abstract

Many empirical studies argue that the inertial behavior of policy rates in industrialized countries can be well explained by a linear partial adjustment version of the Taylor rule. However, the explanatory power of the lagged interest rate has been questioned from various points of view. This paper formally examines a situation in which a central bank has an aversion to frequent policy reversals. Imposing an irreversibility constraint on the control space makes the lagged interest rate a state variable. However, the policy function cannot then be expressed as a partial adjustment form even if the original Taylor rule would be the correct policy function in the absence of the irreversibility constraint. The simulation results reveal that conventional regression tends to falsely support the functionally misspecified partial adjustment model. This implies that the significant role of the lagged interest may simply reflect the central banks' aversion to policy reversal.

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*Department of Economics, Chukyo University, 101-2 Yagoto-honmachi, Showa-ku, Nagoya 466-8666, Japan. E-mail: kteru@mecl.chukyo-u.ac.jp. Tel, fax: +81-52-835-7943. I would like to thank Junya Masuda and two anonymous referees for valuable comments and suggestions. This research was supported by KAKENHI 21243027.

1 Introduction

The gradual adjustment of policy rates has long been deemed as good evidence of a central bank's preference for conservatism. The most popular way to measure the extent of gradualism is to estimate the coefficient on the lagged interest rate in the following Taylor rule:

$$i_t = \psi_i i_{t-1} + (1 - \psi_i)(c_0 + c_1 \pi_t + c_2 y_t),$$

where i_t is the policy rate at time t, and π_t and y_t are inflation and the output gap, respectively. This partial adjustment version of the Taylor rule, which is occasionally called the "generalized Taylor rule", has been heavily used not only in empirical studies but also in theoretical studies, especially in the new-Keynesian DSGE models (e.g., Clarida, Galí and Gertler, 1999, Woodford, 2003a,b).

Many studies also address the issue of why central banks move their policy rates gradually. For example, Castelnuovo and Surico (2004) and Wieland (2006) argue that the presence of model or parameter uncertainty leads central banks to move their policy rates gradually. Woodford (2003a, b) also points out that the inertial behavior of a policy rate can mimic optimal commitment policy. He insists that the history dependency of a policy rate enables the central bank to affect expectations of future interest rates more effectively than in the case of discretionary policies. From a different point of view, Guthrie and Wright (2004) argue that the introduction of both fixed and proportional adjustment costs of changing interest rates can help explain the gradual behavior of interest rates. Intuitively, fixed costs are necessary since otherwise the central bank could change the interest rate at any time. On the other hand, proportional costs are necessary since otherwise the interest rate would be shifted by an unrealistically large degree.[1]

Despite its popularity, the partial adjustment version of the Taylor rule has been occasionally criticized from various viewpoints. Rudebusch (2002) argues that if a central bank smoothes interest rates, then future short-term interest rates must be predicted quite accurately. He shows that the predictability of future short-term rates is not great enough to support interest rate smoothing, insisting that the explanatory power of the lagged interest rate is attributable to the presence of serially correlated omitted variables. Sack (2000) states that the presence of parameter uncertainty would lead the Fed to act less aggressively compared to the optimal policy under certainty. Trehan and Wu (2007) show that failing to include a time-varying equilibrium real interest rate in the estimation equation can exaggerate the degree of interest rate smoothing that is actually occurring. Gorodnichenko and Shapiro (2007) insist that the gradualism of the Fed's monetary policy during the Great Moderation can be seen as reflecting its price-level targeting. They find that once a price-level gap is introduced in the Taylor rule, the significance of the lagged

[1] See Sack and Wieland (2000) for a survey of the literature on interest rate smoothing.

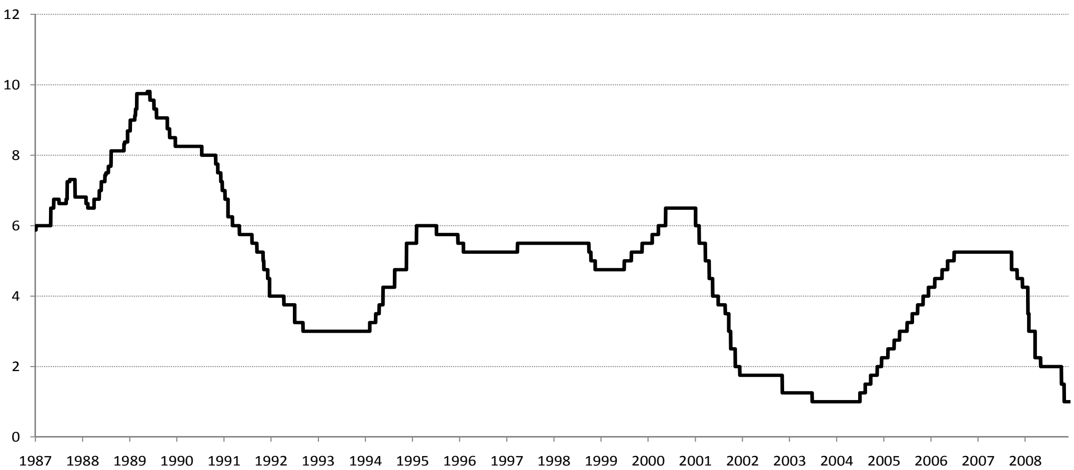

Figure 1: The Federal funds target

interest rate disappears. More recently, Consolo and Favero (2009) reestimate the Taylor rule using GMM, taking into account the possibility of weak instruments. They show that GMM estimation that takes care of the weak instruments problem makes the estimated coefficient on the lagged interest rate significantly smaller than previous studies suggest.

In this paper, I present an alternative explanation of why the partial adjustment model can be incorrect. I consider a situation in which a central bank has an aversion to frequent policy reversals. In practice, central banks apparently try to avoid policy reversals, since reversals signal to the markets that policy shifts may be modified or even neutralized in the near future. Such a possibility of altering policy directions would undermine the central bank's credibility and thus the effectiveness of monetary policy. From 1990 through 2009, the Fed changed the Federal funds target 95 times, and there are only two cases in which the Fed reversed policy direction within two quarters. In fact, the shortest interval between opposite target shifts was about 5 months (Figure 1). This pattern of infrequent policy reversal is inconsistent with the quarterly partial adjustment model which allows for immediate reversals.

Whereas Lowe and Ellis (1997) and BIS (1998) point out the possibility that an aversion to policy reversals would lead the central banks to conservative, I introduce the reversal aversion of central banks more formally. I consider a situation where a central bank faces an irreversibility constraint that prohibits the policy rate from moving in opposite directions within two consecutive periods. This can be interpreted as a special case of the situation where the central bank bears the cost of policy-rate reversals without any restriction on the control space. Thus, as noted by Lowe and Ellis (1997), this discussion is similar to the irreversibility of investment (e.g., Dixit and Pindyck, 1994). The introduction of irreversibility will make the optimal policy less aggressive than would be the case if there was no constraint. This is simply because a larger policy shift would increase the likelihood that the policy rate must be reversed in the next period. In other words, there arises an option value to wait, as is the case with investment irreversibility.

It turns out that the presence of the irreversibility constraint tends to yield a wrong conclusion that the linear partial adjustment model is correct. Obviously, the linear partial adjustment equation is functionally misspecified and omits an important state variable as long as the central bank faces the irreversibility constraint. I also show that the application of English, Nelson and Sack's (2003) specification test, which simultaneously allows for both the partial adjustment term and serial correlations, would not correctly detect the misspecification. This suggests that the widely used partial adjustment model may be falsely accepted since the policy rates under irreversibility are observationally indistinguishable from those under the partial adjustment model.

2 A simple model of gradualism

This section presents a very simple model in which there is no interaction between the central bank's policy action and the desired interest rate, thus allowing us to clarify the pure effect of introducing an irreversibility constraint. Assume that the central bank knows the process of the desired interest rate, which is given as

$$i_t^* = \rho i_{t-1}^* + \varepsilon_t, \quad \rho \in [0, 1), \tag{1}$$

where i_t^* is the current desired interest rate and $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$. i_t^* is called the *desired* interest rate in the sense that the central bank always sets its policy rate, i_t, at i_t^* if there is no restriction on the behavior of the policy rate. However, if the central bank has an aversion to frequent policy reversals, then i_t^* is not necessarily the *optimal* interest rate since the previous policy rate becomes an upper (a lower) bound for the current policy rate when the policy rate was previously lowered (raised).

A central bank that has an aversion to policy reversals not only avoids current policy reversals but also takes into account the influence of its current policy action on the likelihood of future policy reversals. For example, suppose that the desired level of interest rate is currently quite high but likely to decline in the next period. In this situation, an upward shift in the policy rate will increase the probability of policy reversal (i.e., a negative policy shift) in the next period. The central bank may decide to keep the policy rate unchanged if such a future policy reversal is highly likely. Thus, the decision-making of a central bank that tries to avoid frequent policy reversals is intrinsically a dynamic problem and the optimal policy will be forward-looking.

Assume that the central bank has a quadratic return function that takes the maximum value zero when the policy rate is equal to i_t^*. One way to incorporate the central bank's reversal aversion is to directly introduce an irreversibility constraint in the Bellman equation:

$$V(i_{t-1}, \delta_{t-1}, i_t^*) = \max_{i_t \in \Omega_t} \{-(i_t - i_t^*)^2 + \beta E_t V(i_t, \delta_t, i_{t+1}^*)\}, \tag{2}$$

where $V(\cdot)$ is the value function of the central bank, $\delta_t \equiv \text{sgn}(i_t - i_{t-1})$ and β is the discount factor. The control space for i_t is constrained by Ω_t, where

$$\begin{aligned} \Omega_t &= \{i_t | \ i_t \leq i_{t-1}, i_t \in \Omega\} \quad \text{if} \ \ \delta_{t-1} = -1, \\ &= \Omega \quad \text{if} \ \ \delta_{t-1} = 0, \\ &= \{i_t | \ i_t \geq i_{t-1}, i_t \in \Omega\} \quad \text{if} \ \ \delta_{t-1} = 1. \end{aligned} \tag{3}$$

Ω denotes the overall control space. There are three state variables in the value function. The most obvious one is the desired policy rate.[2] In addition, i_{t-1} and δ_{t-1} become state variables since they determine the current control space. It should be noted that the current policy not only affects the one-period return, $-(i_t - i_t^*)^2$, but also determines the next period's state variables i_t and δ_t, which constrain the future control space. It is this forward-looking aspect that creates the possibility that the central bank favors gradualism.

Another possible way to examine the central bank's aversion to policy reversals is to penalize or place a cost of reversals. In this case, the control space is not constrained by the direction of the previous policy shifts, while policy reversals impose penalties or costs on the central bank through the one-period return function. This type of problem setting is called the penalty method or the barrier method, in which case the solution could be obtained by a perturbation method once the constraint is rewritten as $\Delta i_t \Delta i_{t-1} \geq 0$.[3] However, a shortcoming of the penalty method is that the problem is not exactly the same as the corresponding constrained problem unless the size of the penalty is infinite outside the boundary.[4] This property will cause a serious problem since the irreversibility constraint demands arbitrary accuracy around the boundary due to the fact that the next period's control space depends largely on $\text{sgn}(\Delta i_t)$. For this reason, introducing a penalty function or a barrier function is inappropriate for a problem that includes the irreversibility constraint.

[2] It is assumed that i_t^* is observable at the time the central bank sets i_t.

[3] See, for example, Luenberger (2003) for an explanation of the penalty and barrier methods.

[4] den Haan and Wind (2009) also argue that for the perturbation methods to be accurate, the penalty function must not have a singularity, say x^*, since the radius of convergence of the Taylor series is bounded above by $||\bar{x} - x^*||$, where $\bar{x}$ is the point at which the approximation is based (See also Judd 1998, Theorem 6.1.2.). This requires the penalty function or the barrier function to be differentiable at the boundary, but this is done at the expense of solution accuracy around the boundary.

2.1 Stochastic simulation

The introduction of an irreversibility constraint makes the central bank's policy function highly nonlinear, and an analytical solution is thus no longer available even in this otherwise simple linear-quadratic framework. In the following, I solve the dynamic programming numerically using value function iteration.

The baseline parameters and grid sizes are as follows. $\beta = .9$, $\sigma_\varepsilon = .005$. As for the persistence of the desired interest rate, I consider two alternative cases: $\rho = .8$ and 0. $\Omega = [-.03, .03]$, and the grid size for state i_{t-1} is set at .0025.[5] As for i_t^*, the AR(1) process is discretized by the method proposed by Tauchen (1986). The number of nodes for i_t^* is 11. The optimal policy in each iteration is obtained by using a modified golden search algorithm, where the optimal value attained by the standard golden search method is compared with the value under no policy shift (i.e., $i_t = i_{t-1}$). Such a modification is necessary because the standard golden search method often fails to calculate corner solutions.[6] The next period's expected value is approximated using a piece-wise linear spline at points not on the grid.[7] The approximated value function is given by

$$V(\hat{i}_j, \hat{\delta}_k, \hat{i}_l^*) = \max_{\bar{i} \in \Omega_{j,k}} \{-(\bar{i} - \hat{i}_l^*)^2 + \beta \sum_{m=1}^{11} P(l,m) V(\bar{i}, \text{sgn}(\bar{i} - \hat{i}_j), \hat{i}_m^*)\}, \qquad (4)$$

where $\hat{i}_j$, $\hat{\delta}_k$ and $\hat{i}_l^*$ denote the grid points for i_{t-1}, δ_{t-1} and i_t^*, respectively. The current control space for $\bar{i}$ depends on indices j and k since the space is determined by the previous interest rate, $\hat{i}_j$, and the previous increment, $\hat{\delta}_k$. $P(l,m)$ denotes the transition probability from state $\hat{i}_l^*$ to $\hat{i}_m^*$.

Figure 2 illustrates the sample paths of the policy rate and the desired rate. It also shows the sign of policy increments (right scale). The figure shows that there is a large discrepancy between the two interest rates. The point is that the discrepancy is observed not only when the irreversibility constraint is binding but also when it is not. The phenomenon that the policy rate often deviates from the desired rate when the constraint is not binding corresponds to the central bank's gradualism, where it moves the policy rate to a lesser extent than the change in the desired rate. As noted above, this is because a larger shift in the policy rate will increase the probability of the next period's policy reversal.

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[5]The choice of model frequency is not an essential issue in this simple model, so the model's one period can be interpreted either as one quarter or as one year. The value of discount factor β seems lower than the conventional value (such as .99), but this choice is simply for the sake of enhancing convergence speed.

[6]Recall that i_{t-1} becomes an upper or a lower bound of i_t when δ_{t-1} is nonzero. See Miranda and Fackler (2002) for an explanation of the standard golden search algorithm.

[7]The criterion of convergence is $||\frac{V^k(\cdot)-V^{k-1}(\cdot)}{V^{k-1}(\cdot)}||_\infty < 10^{-6}$, where $V^k(\cdot)$ is the value after kth iterations.

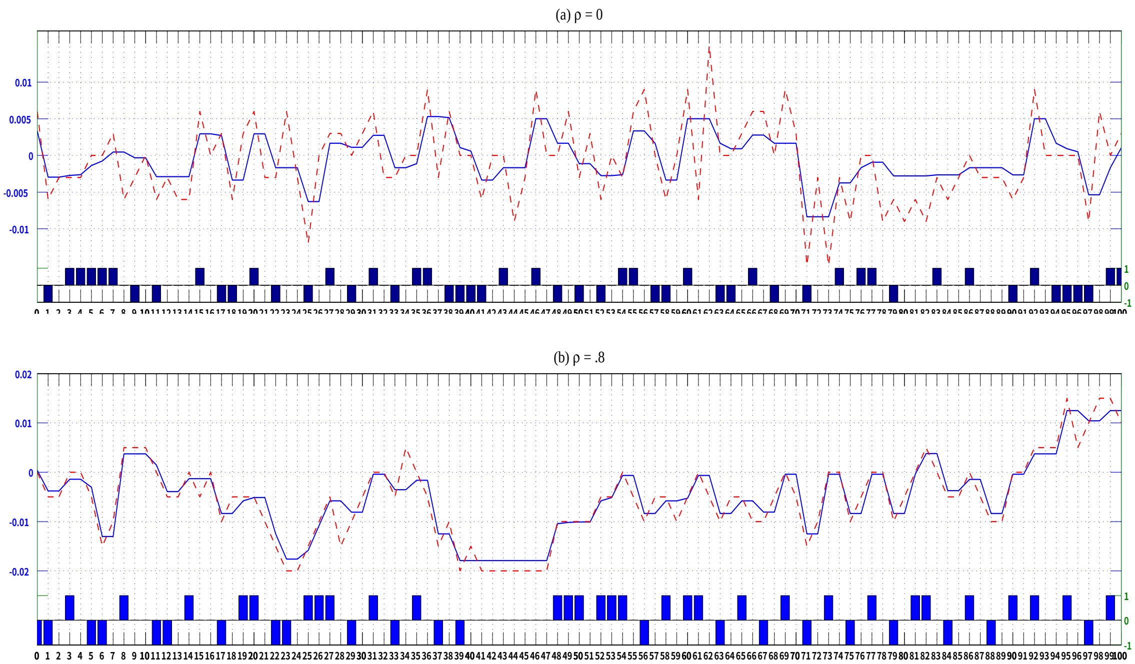

Figure 2: Simulated paths: The simple model

Note: The solid line and the dashed line denote the optimal policy rate under the irreversibility constraint and the desired rate, respectively. The bars are the signs of policy shifts (right scale).

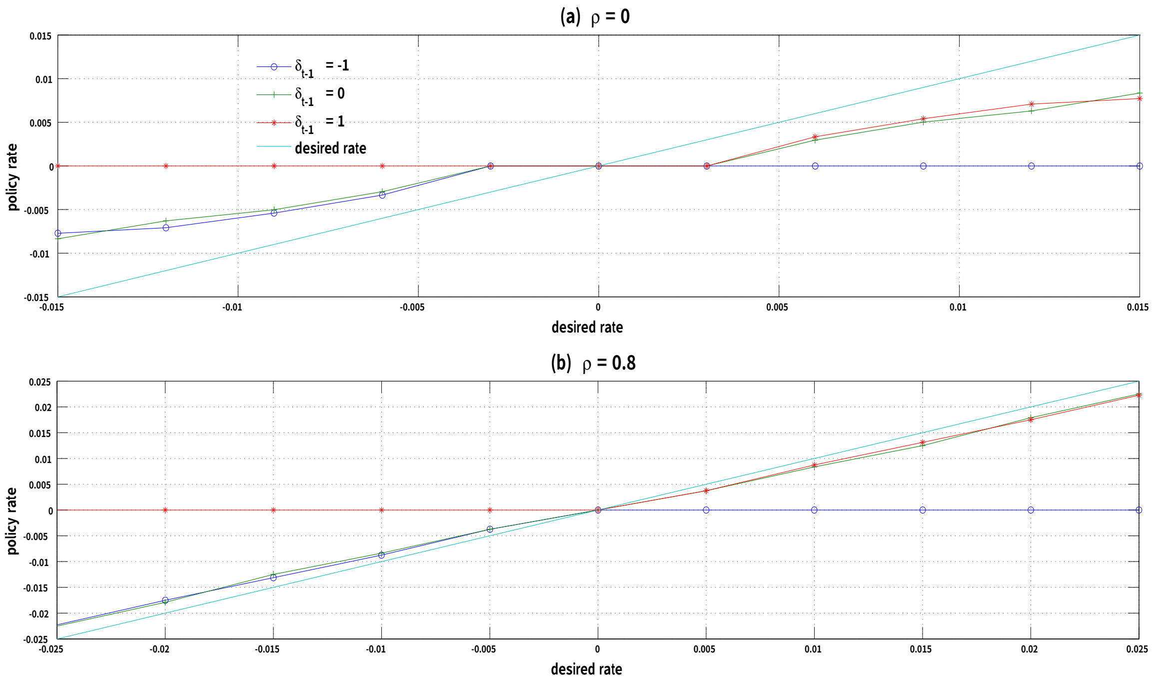

Figure 3: Policy function: The simple model

Another interesting feature is that the size of the discrepancy between the policy rate and the desired rate depends on the degree of persistence, ρ. It turns out that, on average, the discrepancy is wider under $\rho = 0$ than under $\rho = .8$. The intuition is that the central bank follows changes in the desired rate more frequently as the degree of persistence increases since the probability of reversal in the desired rate decreases in ρ. This implies that the extent of gradualism is endogenously determined, depending on the persistence of the desired rate.

Figure 3 shows the policy function when $i_{t-1} = 0$. The two figures reconfirm the two properties shown in the simulated paths: i) the central bank that faces the irreversibility constraint pursues a gradual monetary policy even when the constraint is not binding, and ii) the degree of gradualism depends on the degree of persistence in i_t^*. In particular, Figure 3a reveals that it is optimal to do nothing when the change in the desired rate is sufficiently small and the persistence of the desired rate is sufficiently small. The relationship between the extent of gradualism and the persistence in i_t^* is far from linear.

2.2 Conventional regression: A partial adjustment model

Since the influential work by Taylor (1993), a large number of empirical studies have attempted to estimate a simple policy rule, and many of them supports a linear partial adjustment model of the form:

$$i_t = \rho_i i_{t-1} + (1 - \rho_i) c_{i^*} i_t^* + \xi_t, \tag{5}$$

where ξ_t is a residual term and i_t^* typically represents the original Taylor rule, or the "Taylor rate", that depends only on inflation and an output gap.

Eq.(5) is obviously not a correct policy function in the presence of the irreversibility constraint, while the gradual adjustment of the simulated optimal policy shown in Figure 2 does not seem inconsistent with the partial adjustment model. Even in theoretical studies, it is often *assumed* that the linear partial adjustment model can well capture an actual central bank's conservative behavior, which is called "interest rate smoothing". However, if the central bank's conservativeness stems from an aversion to policy reversals, then such a conventional assumption will not hold true.

One might argue that the partial adjustment model may still describe a central bank's conservatism as a first-order approximation to the possibly nonlinear policy function. Unfortunately, the policy function under the irreversibility constraint is not only nonlinear but also depends on additional state variable δ_{t-1}. Therefore, the standard linear partial adjustment model is functionally misspecified even around the steady state.

In the following I estimate eq.(5), using simulated data obtained by the model presented in the previous subsection. If the estimated values of ρ_i and c_{i^*} are statistically significant, then the possibility arises that the conventional regression of the partial adjustment model may be spurious.

2.3 Estimation results

Since the data used in the previous studies are mostly quarterly and start from the late 1980s, I set the length of simulated data used in the estimation at 100. In doing so, I generated 5000 sets of 200-period-long data, and discarded the initial 99 periods.[8] The distributions of the OLS estimates are shown in Figures 4 and 5.

First, it turns out that the coefficient on the lagged interest rate, ρ_i, is significantly different from 0 in both cases and tends to be larger under $\rho = 0$ than under $\rho = .8$. This difference in the estimated values of ρ_i can be thought of as reflecting the extent of discrepancy between the policy rate and the desired rate. Second, the coefficient on i_t^*, c_{i^*}, is less than 1 in most cases, while c_{i^*} should be unity under the assumption that the linear partial adjustment model is correct.

Figures 4c and 5c also show that the Breusch-Godfrey LM test often fails to reject the null hypothesis that the OLS residuals are serially uncorrelated.[9] Although,

[8]I discarded 99 periods, not 100 periods, since lagged data are needed.

[9]The Durbin-Watson test is inappropriate when the regression contains a lagged dependent variable, because the test statistic will be biased toward a finding of no serial correlation (e.g., Nerlove and Wallis, 1966).

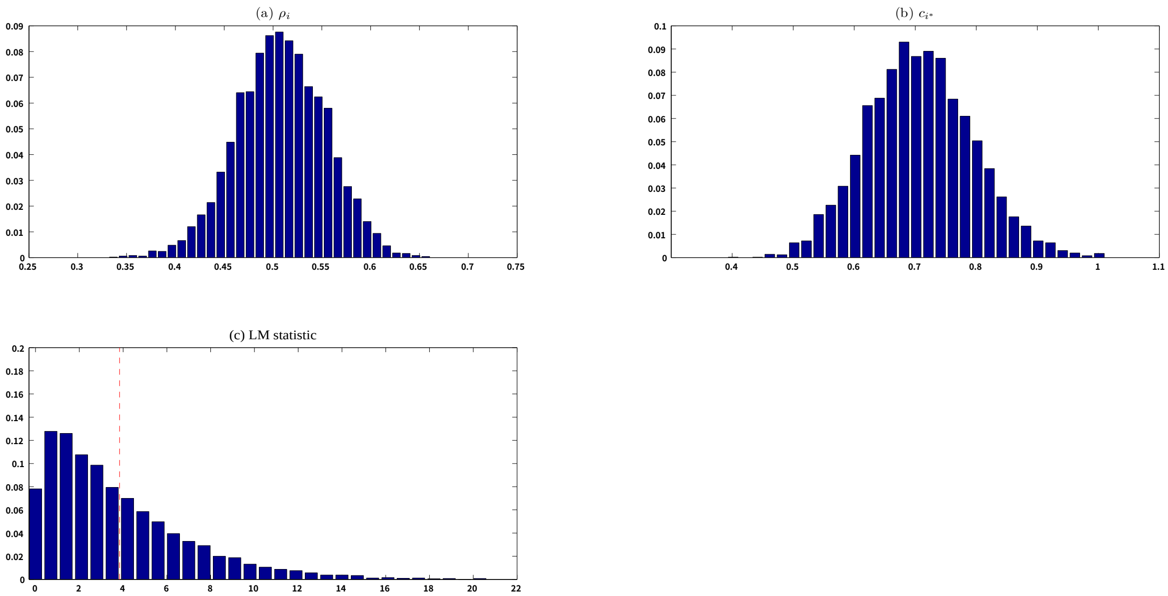

Figure 4: Histogram of the least squares estimates of eq.(5): $\rho = 0$.

Note: Dashed vertical line: the 95% critical value for the Breusch-Godfrey LM test statistic.

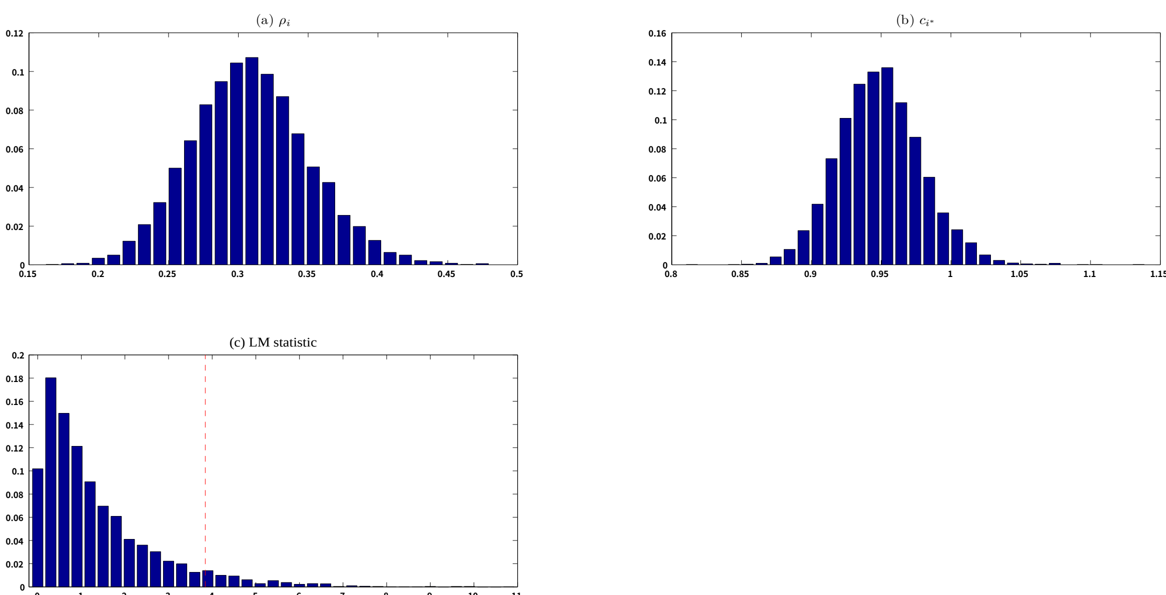

Figure 5: Histogram of the least squares estimates of eq.(5): $\rho = .8$.

Note: Dashed vertical line: the 95% critical value for the Breusch-Godfrey LM test statistic.

in general, the presence of omitted variables often causes a serial correlation of the OLS residuals, the result that serial correlations are not detected does not necessarily mean that the estimated equation correctly contains all the true explanatory variables. Obviously, the estimated policy rule is functionally misspecified in terms of nonlinearity, and a key state variable δ_{t-1} is incorrectly omitted from the model. The Monte Carlo experiments reveal that this type of error is more likely to occur as the desired rate becomes more persistent.

This simple simulation exercise clearly states that the OLS estimation of the functionally misspecified equation can easily lead to an incorrect conclusion that the actual policy rate can be expressed by a linear partial adjustment model. Recall that the above results are obtained under the assumption that the current desired rate, i_t^*, is known with certainty. Many empirical studies focus on correctly estimating the Taylor rate i_t^*, but the conventional regression would be inappropriate even if we have accurate information about the Taylor rate.

3 The Ball-Svensson backward-looking model

3.1 The basic framework

Thus far, I have examined the situation in which a central bank's policy action does not affect the desired interest rate. In practice, however, explanatory variables in the policy rule, typically inflation and an output gap, are inevitably affected by the policy rule itself. This section employs a simple structural model in order to take into account the interaction between the policy rule and its explanatory variables.

The economic structure is expressed by the following Ball-Svensson's model:[10]

$$y_{t+1} = \rho_y y_t - \delta_y(i_t - E_t\pi_{t+1}) + \nu_{t+1}, \quad (6)$$
$$\pi_{t+1} = \gamma_\pi \pi_t + \alpha_\pi y_t + \eta_{t+1}, \quad (7)$$

where y_t is an output gap and π_t is the rate of inflation. ν_{t+1} and η_{t+1} are white noises whose variances are σ_ν^2 and σ_η^2, respectively. Since the expected inflation is expressed as $E_t\pi_{t+1} = \gamma_\pi\pi_t + \alpha_\pi y_t$, the output gap in period $t+1$ leads to

$$y_{t+1} = (\rho_y + \delta_y\alpha_\pi)y_t - \delta_y i_t + \gamma_\pi\delta_y\pi_t + \nu_{t+1}. \quad (8)$$

A virtue of this simple backward model is that the standard Taylor rule is obtained as the optimal policy function. To demonstrate this, assume that the central bank

[10]See Ball (1999) and Svensson (1997). Kato and Nishiyama (2005) investigate the effect of the zero-lower bound on the policy rates within this model. The only difference from their model is that I allow the non-unitary coefficient on π_t, γ_π. The reason for this modification is described below.

has a quadratic one-period return function. The Bellman equation is written as[11]

$$V(y_t, \pi_t) = \max_{i_t} \{-y_t^2 - \lambda \pi_t^2 + \beta E_t V(y_{t+1}, \pi_{t+1})\}. \tag{9}$$

It can be easily shown that the policy function leads to

$$\tilde{i}_t = \gamma_\pi \pi_t + \frac{\rho_y + \delta_y \alpha_\pi}{\delta_y} y_t + \frac{\gamma_\pi \beta \alpha_\pi a}{\delta_y (1 + \beta \alpha_\pi^2 a)} (\gamma_\pi \pi_t + \alpha_\pi y_t), \tag{10}$$

where $a = (-(1 - \lambda\beta\alpha_\pi^2 - \beta\gamma_\pi^2) + \sqrt{(1 - \lambda\beta\alpha_\pi^2 - \beta\gamma_\pi^2)^2 + 4\lambda\beta\alpha_\pi^2})/(2\beta\alpha_\pi^2)$. Thus, in the absence of any restriction on the policy space, the optimal policy is to follow the Taylor rule without the lagged interest term.

Now, let us define the problem under the irreversibility constraint. The Bellman equation is written as

$$V(i_{t-1}, \delta_{t-1}, y_t, \pi_t) = \max_{i_t \in \Omega_t} \{-y_t^2 - \lambda \pi_t^2 + \beta E_t V(i_t, \delta_t, y_{t+1}, \pi_{t+1})\}, \tag{11}$$

where control space Ω_t is defined in the same way as (3).

Since the Taylor rate is equal to $\tilde{i}_t$, the partial adjustment equation to be estimated should be written as

$$\begin{aligned} i_t &= \phi_i i_{t-1} + (1 - \phi_i)\tilde{i}_t + \xi_t \\ &= \phi_i i_{t-1} + (1 - \phi_i)(\phi_\pi \pi_t + \phi_y y_t) + \xi_t. \end{aligned} \tag{12}$$

The coefficients ϕ_i, ϕ_π and ϕ_y will be estimated simultaneously.

3.2 Simulation

The parameter values are as follows: $\lambda = 4$, $\rho_y = .7$, $\delta_y = .3$, $\gamma_\pi = .8$ and $\alpha_\pi = .05$. The parameter values for structural equations are all set at levels smaller than those used in Kato and Nishiyama (2005) for the following reason.[12] Employing larger coefficients increases the possibility of extrapolation when the expected value is calculated. Since extrapolation often undermines the accuracy of value function approximation, I choose relatively small parameter values so as to reduce the frequency of extrapolation. It should be noted that the parameter values can be increased to more empirically plausible values, but there is a trade-off between empirical plausibility and approximation accuracy.

[11] Since i_t does not affect either π_t or y_t, the Bellman equation can be reformulated by employing $E_t y_{t+1}$ and $E_t \pi_{t+1}$ as a control variable and a state, respectively. However, I use eq.(9) in the following since i) the values of π_t and y_t are needed in estimation and ii) the irreversibility constraint should be defined with respect to the policy rate.

[12] I set the discount factor β at .9, whereas it is .6 in Kato and Nishiyama (2005).

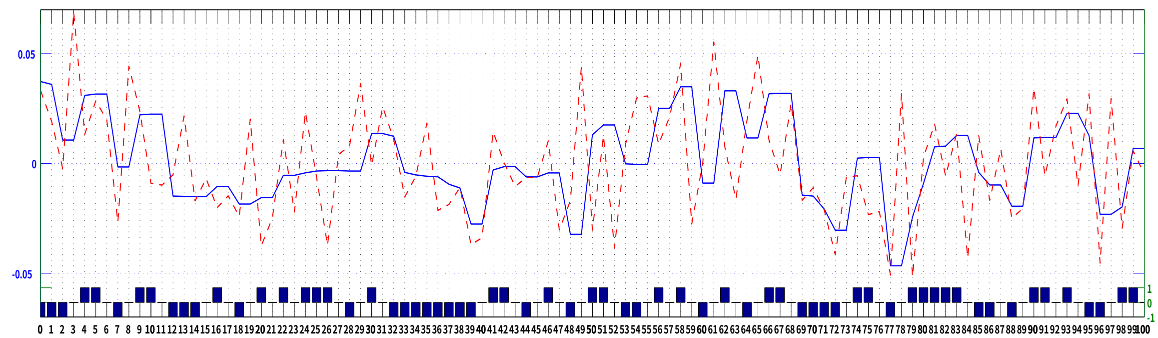

Figure 6: Simulated paths: The backward-looking model

Note: The solid line and the dashed line denote the optimal policy rate under the irreversibility constraint and the desired rate, respectively. The bars are the signs of policy shifts (right scale).

Determination of the overall control space Ω should also be done with great care. Since our focus is on neither the upper nor the lower bound of interest rates, but on the effect of the irreversibility constraint, any influence of restrictions other than the irreversibility constraint must be eliminated. To do this, I choose a sufficiently large control space so that the Taylor rate is covered for all nodes of (y_t, π_t). Given that the optimal policy under the irreversibility constraint is more conservative than the policy under no constraint, this choice of control space will cover all the nodes for the optimal interest rate under irreversibility.

Specifically, the maximum and minimum nodes for i_{t-1} are .24 and -.24, respectively. The maximum and minimum nodes for y_t are .08 and -.08, respectively, while the maximum and minimum nodes for π_t are .04 and -.04, respectively. The grid size is .01 for all states other than δ_{t-1}. The distributions for shocks, ν and η, are obtained in the same way as above, where the variances are both set at $.01^2$. The number of nodes for each shock is 9. With these parameter values, the theoretical values of ϕ_y and ϕ_π are 2.40 and 1.07, respectively. The numerical value function is written as

$$\begin{aligned} V(\hat{i}_j, \hat{\delta}_k,\, \hat{y}_l,\, \hat{\pi}_n) = & \max_{\bar{i} \in \Omega_{j,k}} \{ -\hat{y}_l^2 - \lambda \hat{\pi}_n^2 \\ & + \; \beta \sum_{m=1}^{9} \sum_{s=1}^{9} q_{\nu,m} q_{\eta,s} V(\bar{i}, \mathrm{sgn}(\bar{i} - \hat{i}_j), \hat{y}_+(\hat{y}_l, \hat{\pi}_n, \bar{i}, \hat{\nu}_m), \hat{\pi}_+(\hat{y}_l, \hat{\pi}_n, \hat{\eta}_s)) \}, \end{aligned} \quad (13)$$

where $\hat{y}_+(\hat{y}_l, \hat{\pi}_n, \bar{i}, \hat{\nu}_m) = (\rho_y + \delta_y \alpha_\pi)\hat{y}_l - \delta_y \bar{i} + \gamma_\pi \delta_y \hat{\pi}_n + \hat{\nu}_m$ and $\hat{\pi}_+(\hat{y}_l, \hat{\pi}_n, \hat{\eta}_s) = \gamma_\pi \hat{\pi}_n + \alpha_\pi \hat{y}_l + \hat{\eta}_s$. $q_{\nu,m}$ and $q_{\eta,s}$ are the probabilities that ν_{t+1} and η_{t+1} are in states ν_m and η_s, respectively. The value function is approximated by using cubic splines.

The paths of the optimal interest rate and the Taylor rate, $\tilde{i}_t$, are shown in Figure 6. The fundamental properties are the same as those observed in Figure 2: the policy

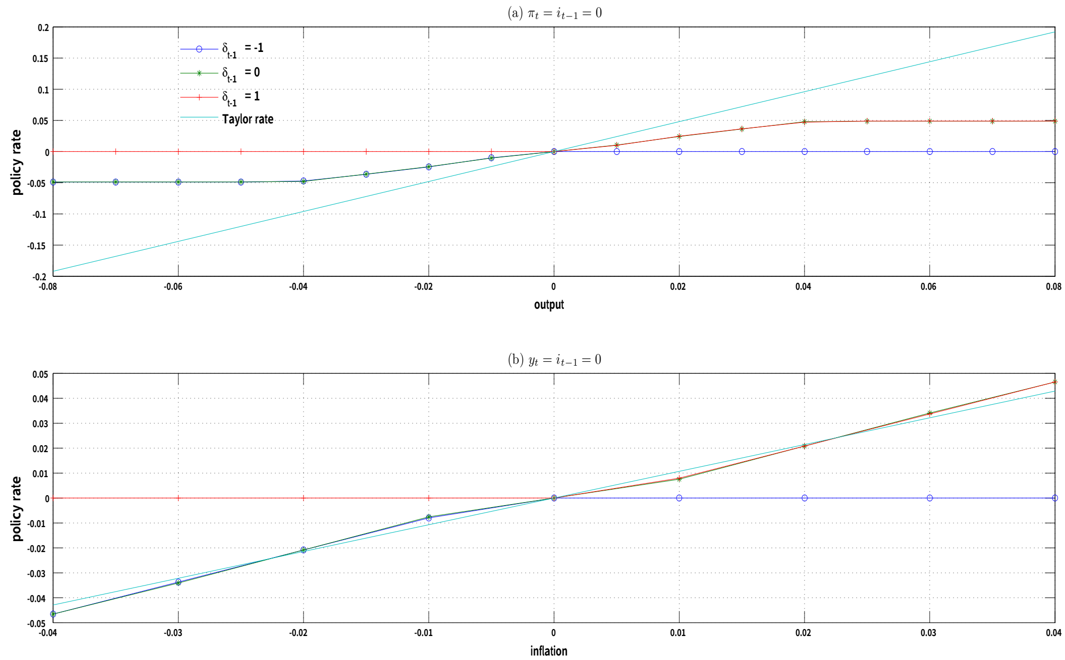

Figure 7: Policy function: The backward-looking model

rate often deviates from the Taylor rate in periods when the irreversibility constraint is not binding. This can also be confirmed by Figure 7, which illustrates policy responses to output and inflation. However, the difference between the Taylor rate and the optimal policy rate is not always significant, depending on the combination of the values of state variables.

Figure 8 shows the histogram of the OLS estimates of the coefficients in (12). The coefficient on the lagged interest rate, ϕ_i, is estimated to be around .5 and is statistically significant. Interestingly, as is the case with the simple model above, the estimated coefficients on inflation and output are both deviated from the corresponding Taylor coefficients. This has an important implication that a central bank's conservatism could not be captured only by the lagged interest rate. While the literature often presumes that the value of ϕ_i represents the degree of policy inertia, the influence of conservatism would also affect the coefficients in the Taylor rate. This implies that conventional estimates of inflation and output coefficients may be biased estimators of the "true" coefficients.

As argued in the previous sections, the optimal policy under the irreversibility constraint may lead to a misleading conclusion that the inertial policy rule can be expressed as a partial adjustment model. The analysis of this section shows that such a misperception could also occur in an environment in which the policy rate and its explanatory variables interact with each other. The coefficient on the lagged interest rate can be viewed as reflecting the central bank's gradualism stemming from

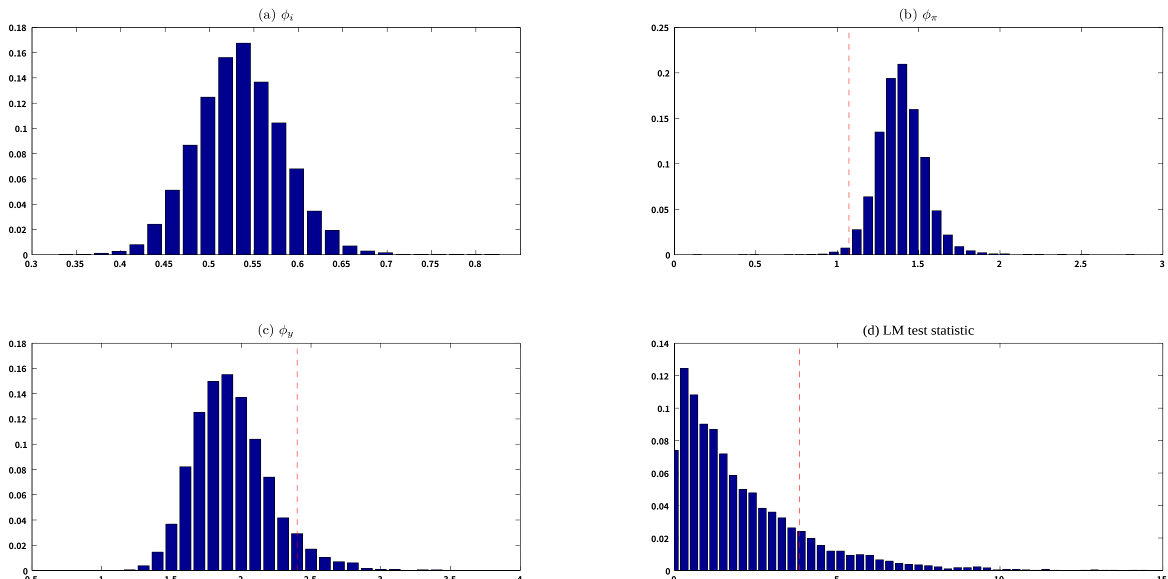

Figure 8: Histogram of the least squares estimates of eq.(12): The backward-looking model.

Note: Dashed vertical lines: theoretical values (b and c), the 95% critical value for the Breusch-Godfrey LM test statistic (d).

the irreversibility constraint, which should be distinguished from the conventional interest rate smoothing expressed by the linear partial adjustment model.

4 The English-Nelson-Sack test

Rudebusch (2002, 2006) argues that the explanatory power of the lagged interest rate in the Taylor rule arises if a serially correlated variable is incorrectly omitted from the estimated equation. If this is the case, the standard estimation of the coefficient on the lagged interest will be biased, and the lagged interest rate will not be able to correctly capture the extent of gradualism.

Based on this argument, English, Nelson and Sack (2003) proposed a nested function that allows for both serially correlated residuals and the lagged interest rate. The equations to be simultaneously taken into account are:

$$i_t = \phi_i i_{t-1} + (1-\phi_i)\tilde{i}_t + u_t, \tag{14}$$

$$u_t = \theta u_{t-1} + \zeta_t, \tag{15}$$

where, as above, $\tilde{i}_t$ is the Taylor rate, u_t is the possibly serially correlated residual, and ζ_t is a white noise. Combining the two equations yields

$$\Delta i_t = (1-\phi_i)\Delta\tilde{i}_t + (1-\phi_i)(1-\theta)(\tilde{i}_{t-1} - i_{t-1}) + \phi_i\theta\Delta i_{t-1} + \zeta_t. \tag{16}$$

Since the presence of serial correlated errors is taken into account, the test for the existence of policy inertia is expected to be free from Rudebusch's critique. English, Nelson and Sack (2003), Gerlach-Kristen (2004) and Castelnuovo (2007) show that the estimate of ϕ_i is still statistically significant.

Let us first consider the simplest model of section 2. Figure 9 illustrates the histograms of the estimates of least squares. It turns out that coefficient ϕ_i always take values greater than zero and less than one. The estimate of θ is also significantly larger than zero. These results are consistent with those obtained by English, Nelson and Sack (2003), Gerlach-Kristen (2004) and Castelnuovo (2007), who insist that the joint formalization of the two hypotheses will be the best approximation to policy rules. However, Figure 10 indicates that the estimate of θ will no longer be significantly different from zero if the desired rate is sufficiently persistent, while the significance of ϕ_i will still be maintained.

In Figures 9d and 10d, I also show the Breusch-Godfrey LM test statistics under the unrestricted regression, which is given as:

$$\Delta i_t = b_1\Delta\tilde{i}_t + b_2(\tilde{i}_{t-1} - i_{t-1}) + b_3\Delta i_{t-1} + \zeta_t. \tag{17}$$

The LM test statistics under the unrestricted regressions are often below the 5% critical value, while the restricted regressions almost always detect serial correlations.

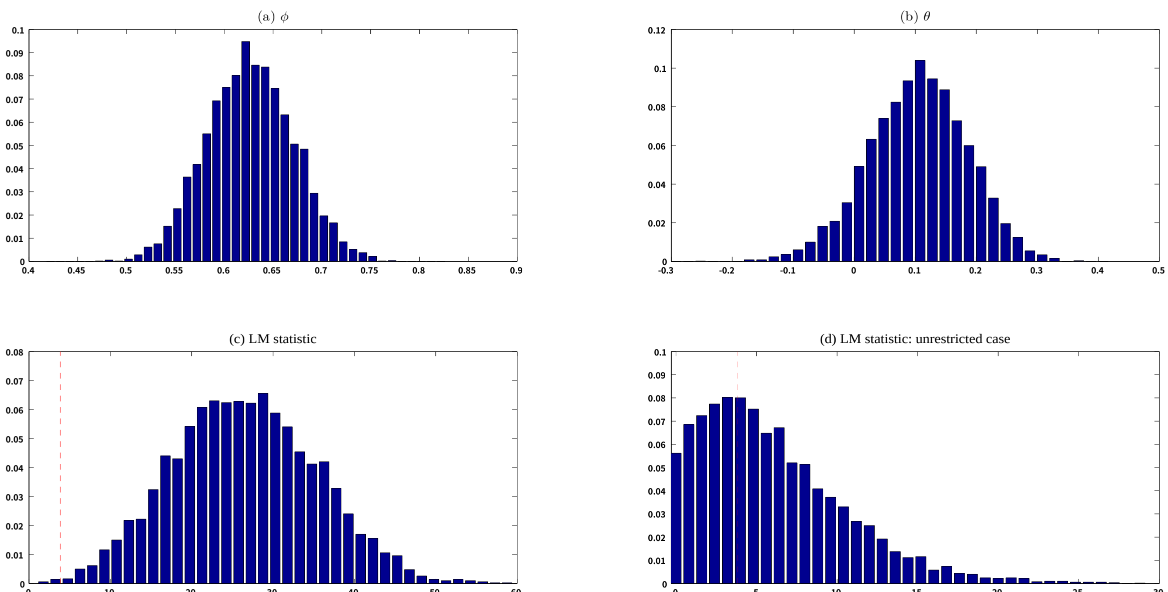

Figure 9: Histogram of the least squares estimates of English-Nelson-Sack equation (16): $\rho = 0$.

Note: Dashed vertical line: the 95% critical value for the Breusch-Godfrey LM test statistic.

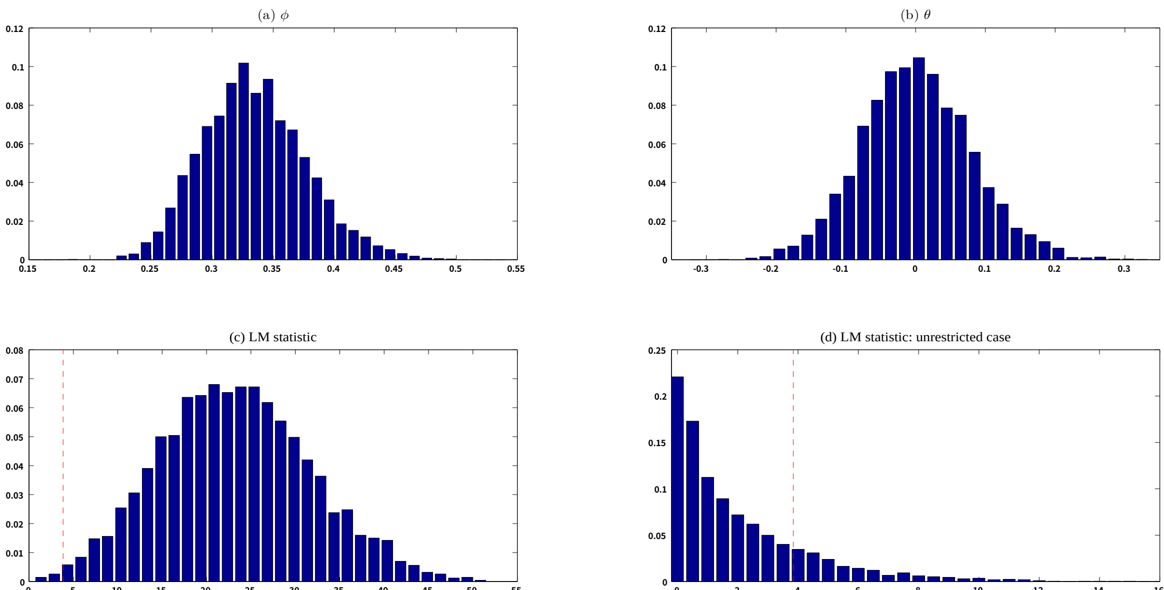

Figure 10: Histogram of the least squares estimates of English-Nelson-Sack equation (16): $\rho = .8$.

Note: Dashed vertical line: the 95% critical value for the Breusch-Godfrey LM test statistic.

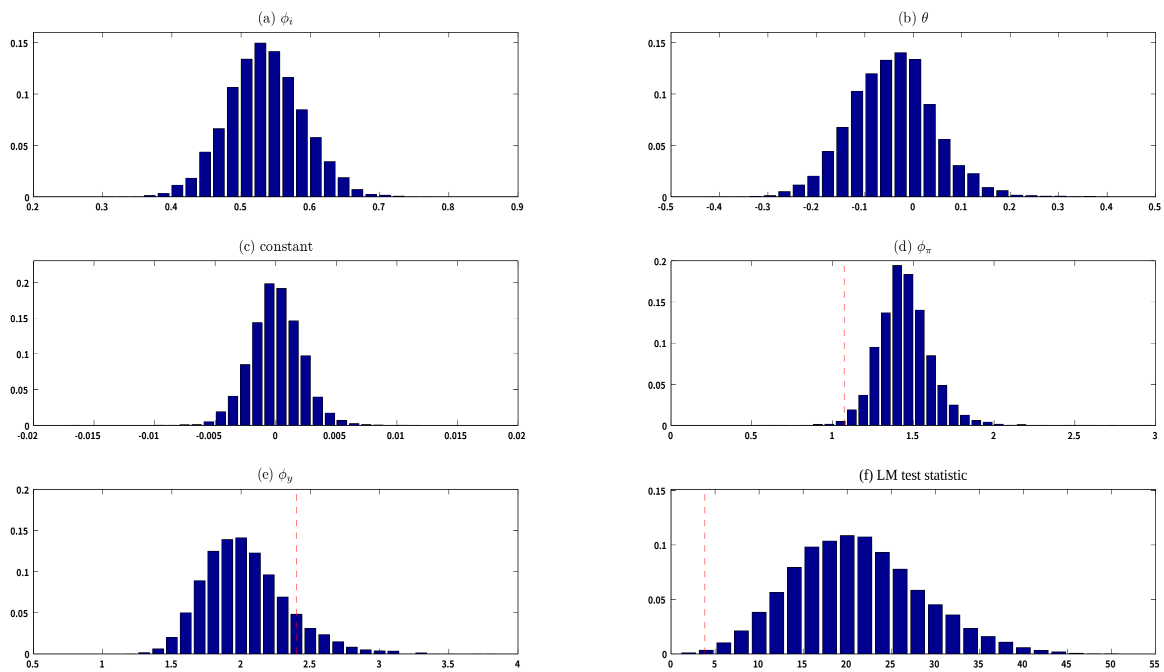

Figure 11: Histogram of the least squares estimates of English-Nelson-Sack equation (16): The backward-looking model.

Note: Dashed vertical lines: theoretical values (d and e), the 95% critical value for the Breusch-Godfrey LM test statistic (f).

This implies that the high values of the LM statistics shown in Figures 9c and 10c do not necessarily stem from the functional misspecification of the nested regression equation but mainly from the nonlinear parameter restriction imposed on (16). The functional misspecification itself, including the presence of omitted variables, could be undetected. In fact, the estimated value of b_3 in the unrestricted regression is significantly greater than zero even in the case of $\rho = .8$, which is inconsistent with the corresponding restricted estimation.[13] Figure 11 shows the estimation results under the backward-looking model. The figures state that the essential results are the same as those illustrated in Figure 10.

These results indicate that the theoretically correct policy function under irreversibility cannot be estimated by (16) even if one allows for the possibility of serially correlated errors. The "statistically significant" estimators arose as a result of the failure to incorporate nonlinearity and state variable δ_{t-1}, which would make the standard least squares estimators inconsistent. Therefore, English, Nelson and Sack's modification to the traditional regression will not work when the central banks' gradualism comes from irreversibility. The results illustrated in Figures 9, 10 and 11 reveal that it would be hard to distinguish between the irreversible monetary policy and the inertial policy under the partial adjustment model just by looking at the estimates of the nested equation eq.(16).

5 Conclusion

This paper shows that the significant role of the lagged coefficient in the estimated Taylor rule may reflect a central bank's aversion to frequent policy reversals since the lagged interest rate becomes a state variable in the presence of an irreversibility constraint. If this is the case, the widely used partial adjustment model is functionally misspecified in terms of nonlinearity and the presence of an omitted variable. This suggests that the degree of policy inertia should be endogenously determined in accordance with the degree of persistence in the desired rate.

An important extension is to introduce forward-looking expectations in AS and IS equations. It is well understood within the linear quadratic framework that making the current policy rate dependent on the lagged policy rates will increase its influence on inflation expectations. However, this idea might not be correct once policy irreversibility is taken into account. One difficulty of introducing expectations in structural equations is to maintain the accuracy of the solution. Considering nonlinear policies under irreversibility will be hard to justify as long as the forward-looking AS and IS equations are obtained by a linearization technique. Ideally, therefore, the model should be solved by a non-local approximation method.

[13] The details of the unrestricted regression are reported in an earlier version of this paper (Kobayashi, 2010).

Another issue is that central banks in practice are considered to be faced with a more relaxed irreversibility constraint than the strict one I introduced in this paper. To introduce a more flexible restriction, the penalty method or the barrier method could be used. The difficulty is that the degree of severity or flexibility of the irreversibility constraint needs to be determined if those more flexible approaches are employed. Therefore, empirical studies are necessary in order to measure the extent of a central bank's aversion to policy reversals.

References

[1] Ball, L., 1999, Efficient rules for monetary policy, International Finance 2, 63-83.

[2] Bank for International Settlements, 1998, *68th Annual Report*, Bank for International Settlements, Basle.

[3] Castelnuovo, E., 2007, Taylor rules and interest rate smoothing in the Euro area, Manchester School 75, 1-16.

[4] Castelnuovo, E., and P. Surico, 2004, Model uncertainty, optimal monetary policy and the preferences of the Fed, Scottish Journal of Political Economy 51, 105-26.

[5] Clarida, R., Galí, J., Gertler, M., 1999, The science of monetary policy: A new Keynesian perspective, Journal of Economic Literature 37, 1661-1707.

[6] Consolo, A., Favero, C.A., 2009, Monetary policy inertia: More a fiction than a fact? Journal of Monetary Economics 56, 900-906.

[7] den Haan, W.J., de Wind, J., 2009, How well-behaved are higher-order perturbation solutions? University of Amsterdam.

[8] Dixit, A. K., Pindyck, R.S., 1994, *Investment under Uncertainty*, Princeton University Press, Princeton, New Jersey.

[9] English, W.B., Nelson, W.R., Sack, B., 2003, Interpreting the significance of the lagged interest rate in estimated monetary policy rules, The B.E. Journal of Macroeconomics: Vol. 3: Iss. 1 (contributions): Article 5.

[10] Gerlach-Kristen, P., 2004, Interest-rate smoothing: Monetary policy inertia or unobserved variables? The B.E. Journal of Macroeconomics: Vol. 4: Iss. 1 (contributions): Article 3.

[11] Gorodnichenko, Y. and M. Shapiro, 2007, Monetary policy when potential output is uncertain: Understanding the growth gamble of the 1990's, Journal of Monetary Economics 54, 1132-62.

[12] Guthrie, G. and J. Wright, 2004, The optimal design of interest rate changes, Journal of Money, Credit and Banking 36, 115-37.

[13] Judd, K.L., 1998, *Numerical Methods in Economics*, MIT Press, Cambridge, Massachusetts.

[14] Kato, R., Nishiyama, S. 2005, Optimal monetary policy when interest rates are bounded at zero, Journal of Economic Dynamics and Control 29, 97-133.

[15] Kobayashi, T., 2010, Policy irreversibility and interest rate smoothing, MPRA paper no. 19931, University Library of Munich.

[16] Lowe, P., Ellis, L., 1997, The smoothing of official interest rates, in Phillip Lowe ed. *Monetary Policy and Inflation Targeting: Proceeding of a Conference*, Reserve Bank of Australia, 286-312.

[17] Luenberger, D.G., 2003, *Linear and Nonlinear Programming, Second Edition*, Kluwer Academic Publishers, Norwell, Massachusetts.

[18] Miranda, M.J., Fackler, P. L., 2002, *Applied Computational Economics and Finance*, MIT Press, Cambridge, Massachusetts.

[19] Nerlove, M., Wallis, K., 1966, Use of the Durbin-Watson statistic in inappropriate situations, Econometrica 34, 235-38.

[20] Rudebusch, G. D., 2002, Term structure evidence on interest rate smoothing and monetary policy inertia, Journal of Monetary Economics 49, 1161-87.

[21] Rudebusch, G. D., 2006, Monetary policy inertia: Fact or fiction? International Journal of Central Banking 2, 85-135.

[22] Sack, B., 2000, Does the Fed act gradually? A VAR analysis, Journal of Monetary Economics 46, 229-56.

[23] Sack, B., Wieland, V., 2000, Interest rate smoothing and optimal monetary policy: a review of recent empirical evidence, Journal of Economics and Business 52, 205-228.

[24] Svensson, L.E.O., 1997, Inflation forecast targeting: Implementing and monitoring inflation targets, European Economic Review 41, 1111-46.

[25] Tauchen, G., 1986, Finite-state Markov chain approximations to univariate and vector autoregressions, Economics Letters 20, 177-181.

[26] Taylor, J.B., 1993, Discretion versus policy rules in practice, Carnegie-Rochester Conference Series on Public Policy 39, 195-214.

[27] Trehan, B., Wu, T., 2007, Time-varying equilibrium real rates and monetary policy analysis, Journal of Economic Dynamics and Control 31, 1584-1609.

[28] Wieland, V., 2006, Monetary policy and uncertainty about the natural unemployment rate: Brainard-style conservatism versus experimental activism, The B.E. Journal of Macroeconomics: Vol. 6: Iss. 1 (advances), Article 1.

[29] Woodford, M., 2003a, *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press, Princeton, New Jersey.

[30] Woodford, M., 2003b, Optimal monetary policy inertia, Review of Economic Studies 70, 861-86.